

2021 John O'Bryan Mathematics Competition
5-Person Team Test

Abbreviated Instructions: Answer each of the following questions **using separate sheet(s) of paper**. Questions will not be scored without the following two things:

- Place your team code in the upper right corner of each page that will be turned in

[This area contains numerous horizontal lines, likely representing a ruled page or multiple sheets of paper. There are some faint, illegible markings and a small handwritten-style 'k =' followed by a plus sign '+' in the lower portion of the page.]

5. The integers from 1 to n are written in a row. The same numbers are written under them in a different order (see below). Note that there are no repeated numbers in a row.

$$\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 1 & 5 & 3 \end{array}$$

~~Notice that the sum of numbers in the middle three columns is a perfect square.~~

=

6. The side BC of an equilateral triangle ABC is divided into three equal parts by the points K and L with K closer to B . The point M divides the side AC in the ratio

$$AM : MC = 1 : 2. \quad + \quad =$$

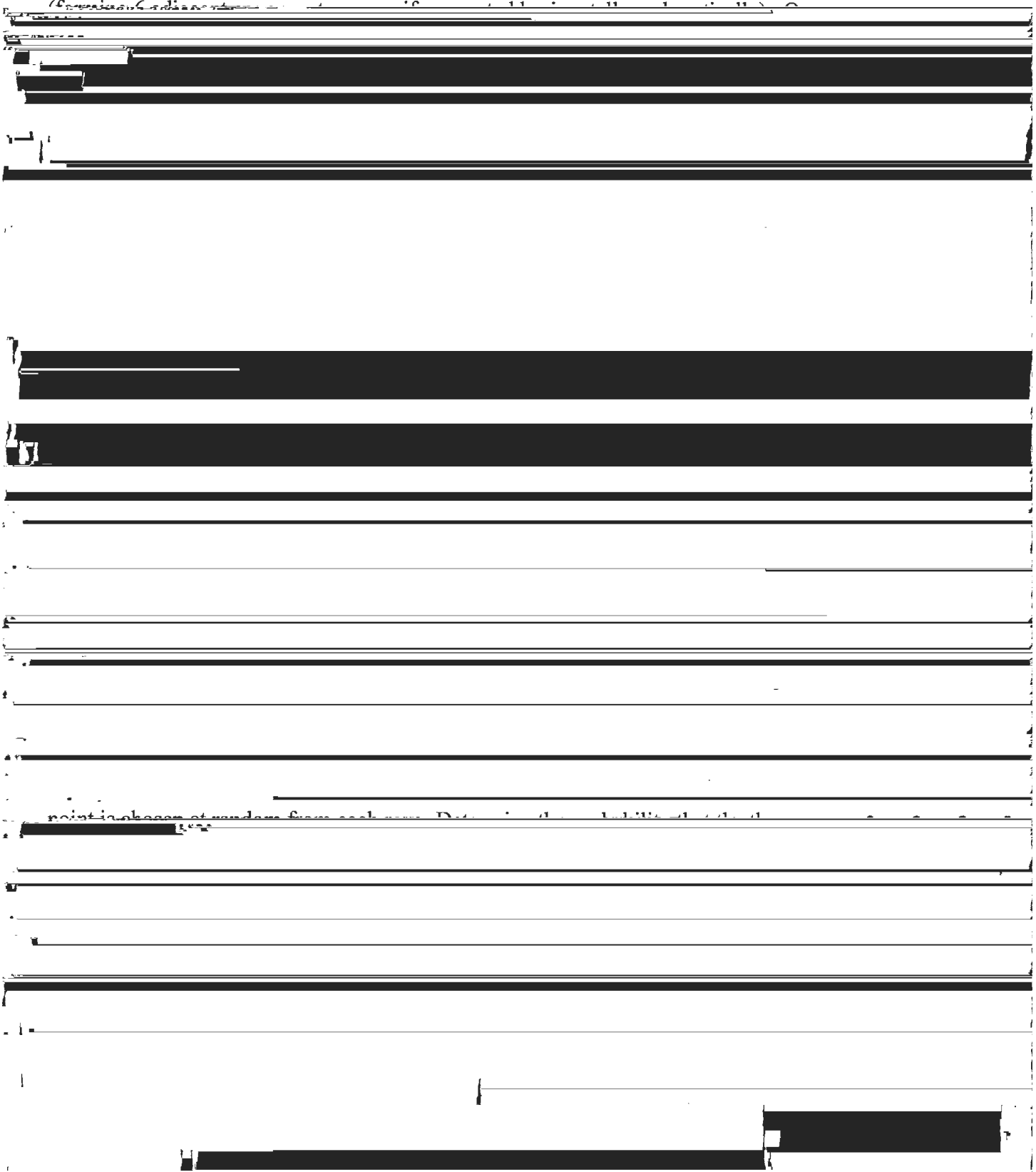
2021 John O'Brvan Mathematical Competition

Freshman-Sophomore Individual Test

Directions: Please answer all questions on the answer sheet provided. All answers must be written in the spaces provided.

9. The rational expression $\frac{\left(3 - \frac{2}{1-x}\right)}{\left(\frac{3}{x-1} - 1\right)} = \frac{kx+w}{p+qx}$ where $k, w, p,$ and q are integers with $k > 0$. Determine the sum $(k+w+p+q)$.
10. \overline{AC} is the diameter of Circle O . Point B lies on the circle such that $\angle BAC = 60^\circ$ and $AB = 10$. Determine the exact length of the radius of Circle O .

11. An array of 12 points is set up in 3 evenly spaced rows and 4 evenly spaced columns



**2021 John O'Bryan Mathematical Competition
Freshman/Sophomore Individual Test**

Note: All answers must be written legibly in the correct blanks on the answer sheet and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has 45 seconds to solve.

1.

11.

3.

13.

4.

14.

5.

15.

6.

16.

7.

17.

8.

18.

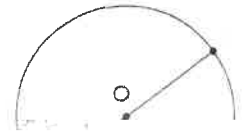
9.

19.

10.

20.

2021 John O'Bryan Mathematical Competition
Junior-Senior Individual Test



B

in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of

13. Let $f(x) = \sqrt{a}x^2 + \sqrt{ab}x + b\sqrt{a}$ with $a > 0$ and $b > 0$. Determine the minimum value for $f(x)$ in terms of a and b .
14. Trailing zeros are the zeros to the right of the last non-zero digit in the integer representation of a number read from left to right. How many trailing zeros does the number $n = 2021!$ have?
15. The triangle shown on the right, $(x + y + z) = 2[\sqrt{k} + \sqrt{w} + \sqrt{p}]$ in simplified and reduced radical form with $k, w,$ and p positive integers. Determine the sum $(k + w + p)$.

16. An array of 12 points is set up in 3 evenly spaced rows and 4 evenly spaced columns.



(forming 6 adjacent congruent squares if connected horizontally and vertically). One point is chosen at random from each row. Determine the probability the 3 points chosen

2

B(26,0)

Name: _____

Team Code: _____

**2021 John O'Bryan Mathematical Competition
Junior/Senior Individual Test**

Note: All answers must be written legibly in the correct blanks on the answer sheet and in simplest form.

Exact answers are to be given unless otherwise specified in the question. No units of measurement.

1. _____

11. _____

2. _____

12. _____

3. _____

13. _____

4. _____

14. _____

5. _____

15. _____

6. _____

16. _____

7. _____

17. _____

8. _____

18. _____

9. _____

19. _____

10. _____

20. _____

2021 John O'Bryan Mathematical Competition
Questions for the Two-Person Speed Event

*****Calculators may not be used on the first four questions*****

1. Let $x = \frac{7}{5} - \frac{0}{-2} = 158$ and $y = \frac{\log_4(27)}{\log_2(9)}$ Find the product xy .

2. Let 36, k , and 20 in that order be the terms in an arithmetic sequence. Let 12, w , and 48 in that order be terms in a geometric sequence with $w > 0$. Then the terms k , w , and p , in that order

3. The point P is located on the x -axis and the point O is located on the y -axis. Each point is 5 units

Names:

School:

2021 John O'Bryan Mathematical Competition
Answers for the Two-Person Speed Event

Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless

1. _____

2. _____

3. _____

4. _____

6. _____

7. _____

8. _____

SCORE

T1. _____

T2. _____

2021 John O'Bryan Mathematics Competition
5-Person Team Test

1. (a) Find prime numbers $p < q < r < s$ satisfying $p^q \cdot r \cdot s = 12008$ and $q \cdot r^p = 1083$

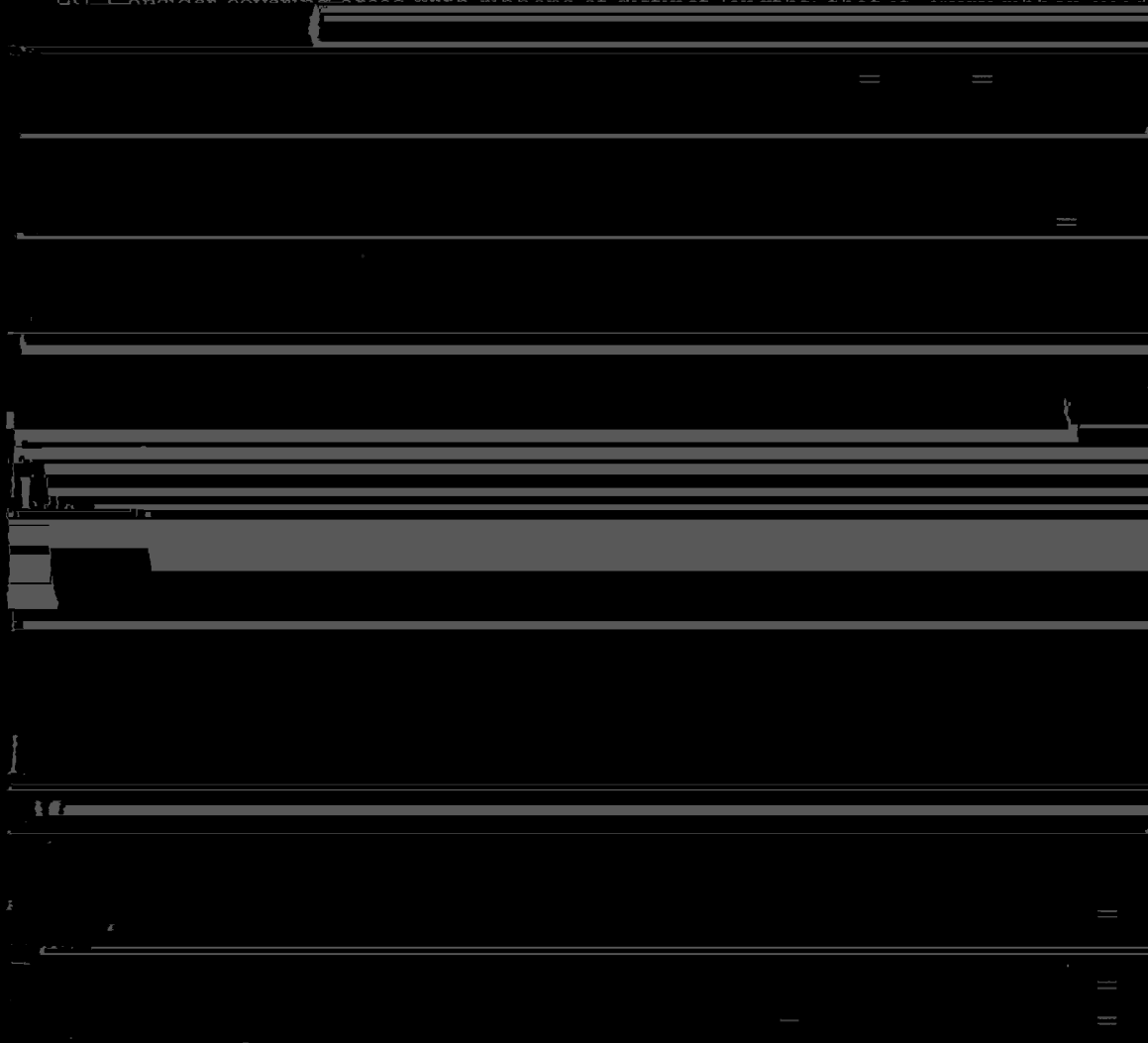
Solution: Since 12008 is even, 2 is a factor: factoring 2s gives $12008 = 2^3 \cdot 1501$.

which means $p = 2$ and $q = 3$. Since the sum of digits of 1083 is divisible by 3,
1083 is divisible by 3. Factoring 3s gives $1083 = 3 \cdot 361$.

$361 = r^2$ implies $r = 19$. Hence $s = \frac{1501}{19} = 79$. Therefore $p = 2, q = 3, r = 19$,
and $s = 79$ giving $12008 = 2^3 \cdot 19 \cdot 79$ and $1083 = 3 \cdot 19^2$.

2. A ribbon of length k is a rectangle with dimensions $1 \times k$ for some natural number k .

(a) Consider covering areas with ribbons of distinct lengths that is, no ribbon is used



to cover the area, has a different length. What is the maximum area that can be

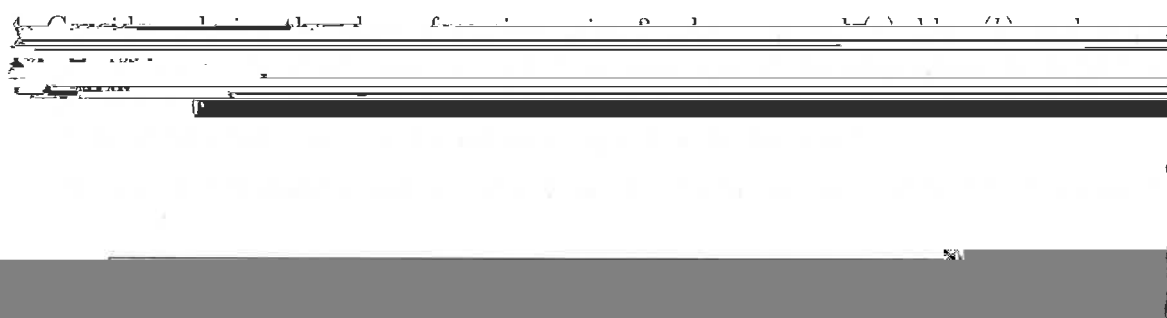
3 (a) For real numbers x and a show that $|x + a| \leq |x| + |a|$. (Hint: if $a > 0$ with

1

2

$$\leq |x + a - z| + |x - a + z| + |x + a + z|$$

3



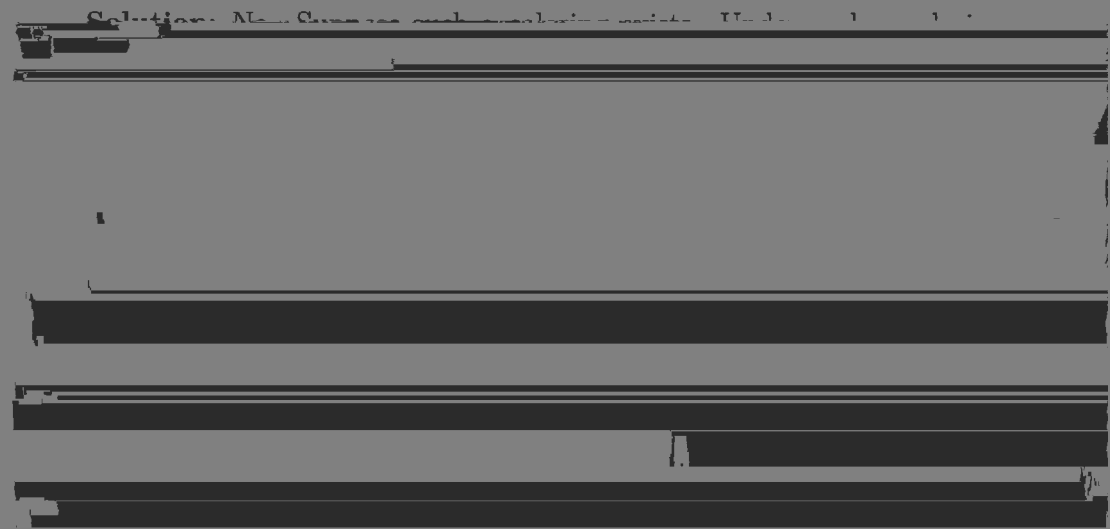
r
r-----
b g

(b) Can a hexagonal prism be colored so that every face and vertex see all 3 colors?

Solution: Yes.

 r
 b g b
 r r
g g r g g
 b g b
 r r

(c) Can a rectangular prism be colored so that every face and vertex see all 3 colors?



The integers from 1 to n are written in a row. The same numbers are written in a second row beneath the first row.

1	2	3	4	5
4	2	1	5	3

Notice that the sum of numbers in the middle three columns is a perfect square. Is it possible that the sums of columns are all perfect squares?

[Redacted area]

(a) $n \equiv 9$?

Solution: Yes. The sum of the number 9 and the number beneath it lies between 10 and 18. Since 16 is the only perfect square in this interval, the number under 9 must be 7. Similarly, the number 7 must be written above the number 9 on the second row. Similarly, the numbers under 4, 5, and 6 must be

[Redacted area]

6. The side BC of an equilateral triangle ABC is divided into three equal parts by the points K and L with K closer to B . The point M divides the side AC in the ratio

[Redacted area containing a diagram and handwritten work for problem 6. The diagram shows an equilateral triangle with side BC divided into three equal segments by points K and L, and side AC divided by point M. The work area contains several lines of text and mathematical symbols, including the word 'Solution' and some equations, but they are mostly obscured by heavy black redaction bars.]

Name: ANSWERS

Team Code:

2021 John O'Brien Mathematical Competition

3.

4.5

Must be this decimal.

13.

60

32

(10,17)

Must be this ordered pair

1 and $-\frac{1}{3}$

Must have both answers in either order

$\frac{2}{15}$

Must be this reduced fraction.

Must be this

$\frac{2636}{495}$

Must be this improper fraction.

$2\sqrt{13}$

Must be this radical expression

8.

276

Name:

ANSWERS

Team Code:

2021 John Q. Brown Mathematical Competition

- | | | | | | |
|-----|--------------|-----------------------------------|-----|--------------------------------|--------------------------------|
| 1. | 2 | | 11. | $-24/25$ | Must be this reduced fraction. |
| 2. | $2\sqrt{13}$ | Must be this radical expression. | 12. | 325 | |
| 3. | 96 | | 13. | $\frac{3}{4}b\sqrt{a}$ | Must be this exact answer. |
| 4. | 2 | | 14. | 503 | |
| 5. | -6 | | 15. | 41 | |
| 6. | $5/6$ | Must be this reduced fraction. | 16. | $1/8$ | Must be this reduced fraction. |
| 7. | $5/3$ | Must be this <input type="text"/> | | 406 | |
| 8. | $-1/2$ | Must be this reduced fraction. | 18. | $1/5$ | Must be this reduced fraction. |
| 9. | 137 | | 19. | 88.47 | Must be this exact decimal. |
| 10. | 2 | | 20. | $\left(\frac{739}{110}\right)$ | |

Names: _____

School: _____

2021 John O'Bryan Mathematical Competition
Answers for the Two-Person Speed Event

Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

SCORE

Calculators are not allowed to be used on the first four questions!

1.

6

2.

20

3.

10

4.

$6 + 3\sqrt{2} + 12\sqrt{3}$

Terms may be in any order

5.

56

6.

17

7.

256

8.

14.4

Must be exactly this decimal.

T1.

0

T2.

50

This competition consists of eight competitive rounds. Correct answers will receive the following scores:

1st: 7 points

2nd: 5 points

All Others: 3 points

There is a three minute time limit on each round. You may submit only one answer each round. To submit your answer, fold this sheet **lengthwise** and hold it high in the air so that a proctor may check your answer.

SCORE